

Strong power and subexponential laws for an ordered list of trajectories of a markov chain

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Abstract

© 2014, International Linear Algebra Society. All rights reserved. Consider a homogeneous Markov chain with discrete time and with a finite set of states E_0, \dots, E_n such that the state E_0 is absorbing and states E_1, \dots, E_n are nonrecurrent. The frequencies of trajectories in this chain are studied in this paper, i.e., “words” composed of symbols E_1, \dots, E_n ending with the “space” E_0 . Order the words according to their probabilities; denote by $p(t)$ the probability of the t th word in this list. As was proved recently, in the case of an infinite list of words, in the dependence of the topology of the graph of the Markov chain, there exists either the limit $\ln p(t)/\ln t$ as $t \rightarrow \infty$ or that of $\ln p(t)/t^{1/D}$, where $D \in \mathbb{N}$ (weak power and subexponential laws). As appeared, in the latter case the decreasing order of the function $p(t)$ is always subexponential (the strong subexponential law). In the first case, this paper describes necessary and sufficient conditions of the power order (the strong power law). These conditions are fulfilled, in particular, if the graph of the Markov chain that corresponds to states E_1, \dots, E_n is strongly connected.

Keywords

Directed graphs, Markov chains, Strong power laws, Substochastic matrices